Online Appendix: The Nonlinear Effects of Uncertainty Shocks

A Time-Varying Volatility

An additional way to think about uncertainty is through its effect on the volatility of the macroeconomic variables. To incorporate this potential effect of uncertainty, we allow uncertainty-driven stochastic volatility in the reduced-form variance-covariance matrix. We make a particular assumption about the form of the stochastic volatility in order to simplify the estimation. In particular, we assume that the change in volatility over time affects all of the variables equally—the so-called common stochastic volatility described in Carriero, Clark and Marcellino (2015). Specifically, we assume that the reduced-form errors are distributed $\varepsilon_t \sim N(\mathbf{0}, \Omega_t)$, where Ω_t will take a particular form such that the stacked vector of innovations, $\boldsymbol{\varepsilon} = (\varepsilon_1, ..., \varepsilon_T)'$, has a Kronecker stucture:

$$vec(\boldsymbol{\varepsilon}) \sim N(0, \Omega \otimes S),$$

where Ω is the equivalent nonstochastic variance and S is a a $T \times T$ matrix that governs the stochastic nature of the variance. Notice that if $S = I_T$, we return the original VAR with constant volatility.

The Kronecker structure allows a myriad of alternative variance specifications [see Chan (2017)]. In our case, S is a diagonal matrix with representative element $s_t = \exp(h_t)$, where h_t governs the volatility such that $\varepsilon_t \sim N(\mathbf{0}, \exp(h_t)\Omega)$. We assume that the log volatility follows an AR(1) process which is also subject to uncertainty shocks via:

$$h_t = \rho h_{t-1} + C(L) Z_t + u_t, \tag{1}$$

where $u_{nt} \sim N(0, \sigma^2)$ and $|\rho_n| < 1$. Notice that the effect of h_t is common across all of the variables in the VAR.

The model with stochastic volatility is a bit more complicated to estimate and requires three additional steps to draw the evolving path of the volatilities and the parameters that govern the path. The three additional steps draw (i) the latent process governing the stochastic volatilities, $\mathbf{h} = \{h_t\}_{t=1}^T$; (ii) the parameters of the stochastic volatiity equation, $\{\rho, C(L)\}$; and (iii) the variance of the stochastic volatility innovation, σ^2.

Conditional on $\mathbf{h}$, the last two blocks are standard, normal–inverse-Gamma draws. The first additional block of the sampler is the draw of $\mathbf{h}$, the latent process governing the stochastic volatility. While there are a number of methods to draw the latent, we utilize the algorithm suggested by Chan (2017), who argues that the prior is normal and the likelihood is approximately normal. Chan suggests employing an MH step to draw $\mathbf{h}$, where the proposal is drawn from

$$\mathbf{h} \sim N\left(K_h^{-1} k_h, K_h^{-1}\right),$$

where

$$K_h = H_\phi' \Sigma_h^{-1} H_\phi + G;$$

$$k_h = f + G\widetilde{\mathbf{h}} + H_\phi' \Sigma_h^{-1} H_\phi \delta_h;$$

H_ϕ defines the state space characterizing the evolution of h_t, $H_\phi \mathbf{h} = \delta_h + \mathbf{u}$; and $\Sigma_h = diag\left(\sigma^2 / \left(1 - \sigma^2\right), \sigma^2, ..., \sigma^2\right)$. The vector f collects the derivatives of the log likelihood evalutated at $\widetilde{\mathbf{h}}$ and G is the negative Hessian evaluated at the same point. The point $\widetilde{\mathbf{h}}$ is, in principle, arbitrary but is generally taken as the mode of $p\left(\mathbf{h}|\mathbf{Y}, \Psi_{-\mathbf{h}}\right)$.

Figure 1 plots the posterior mean estimate of h_t (solid line, left axis), the factor that governs the common stochastic volatility across all variables in the VAR. Intuitively, since the uncertainty index directly affects the log volatility series, h_t resembles the time series for Z_t over the sample (dashed line, right axis). Volatility is high early in the sample, during all three recessions, and again for a stretch after the Great Recession.

One of the advantages of common stochastic volatility in the linear VAR is that h_t scales the size of the shock. Thus, one can estimate the impulse response to a shock in the standard way using the constant portion of the variance-covariance matrix, Ω. The response in any period would be a scalar multiple of the baseline response. In the nonlinear model, an increase in the variance of the uncertainty shock can affect the probability that uncertainty rises past the threshold. In order to estimate the impulse response, we must again use Monte Carlo methods. In addition to sampling from the histories and then generating future values of the ε_t's, we must also sample from the volatilities and generate future values of the u_t's. To maintain consistency with the sampled histories, we use the values of the volatilities that correspond to the dates that we draw for the histories.

The posterior median impulse responses from the stochastic volatility max uncertainty VAR with $b^{xz}(L) = 0$ are plotted in Figure 2, along with the posterior median responses from the baseline model. For comparison, we differentiate between responses under scenario 1 for the baseline (solid line) and the stochastic volatility case (dots). Additionally, we differentiate between responses under scenario 2 for the baseline (dashed line) and with stochastic volatility (open circles). We find little difference from the responses produced by the homoskedastic VAR. An increase in uncertainty creates recessionary effects for all variables in the VAR and the contraction is larger when uncertainty has recently hit a local max. Since h_t scales the effect of the shock, at least at the median, the responses from the model with stochastic volatility are all slightly smaller in magnitude than those produced in the homoskedastic case.

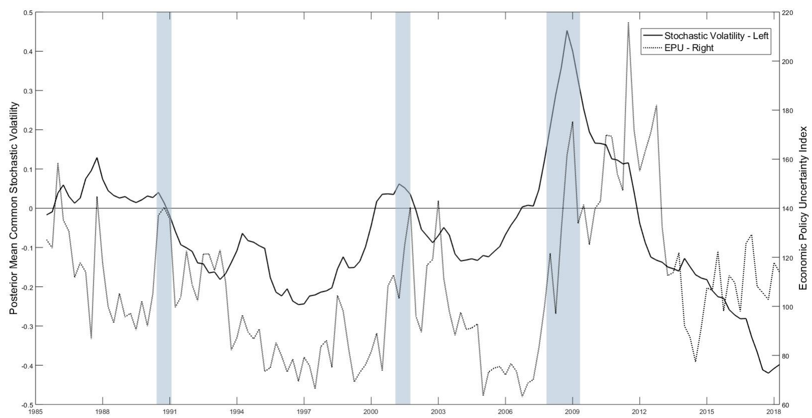

Figure 1: Posterior mean of common stochatic volatility series (solid line - left axis) with the Baker, Bloom, and Davis (2016) Economic Policy Uncertainty Index (dashed line - right axis)

B Financial Stress Channels

A number of papers have suggested that macroeconomic uncertainty and financial uncertainty propagate differently [see Carriero, Clark, and Marcellino (2018)]. Others have argued that financial channels are important amplification channels [see Gilchrist, Sims, and Zakrajšek (2014) and Popp and Zhang (2016)]. While we have a measure of monetary policy in our model, we do not have a financial market channel which can act as an amplification mechanism. In what follows, we include a measure of financial market conditions to see whether deteriorating financial conditions can amplify uncertainty shocks.

We include the National Financial Conditions Index (NFCI) computed by the Federal Reserve Bank of Chicago as an additional variable in the VAR. The NFCI is constructed by extracting a common dynamic factor from 105 measures of financial activity. These series account for conditions in money markets, debt and equity markets, and banking systems. The factor is identified in such a way that positive values indicate tighter-than-average financial conditions and negative values suggest looser conditions. When including the NFCI last in the VAR, after the monetary instrument and a long-term interest rate, we find that increases in uncertainty produce a tightening of overall financial conditions as shownin Figure 3. The effect is slightly larger when uncertainty has recently

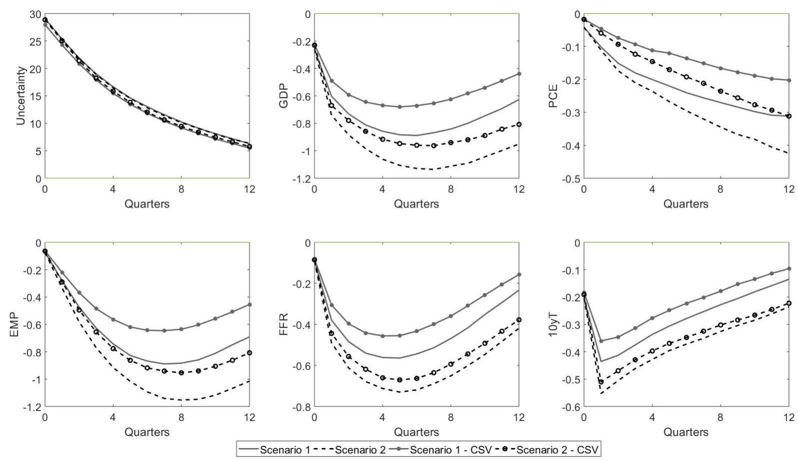

Figure 2: Comparison of impulse responses of GDP from the benchmark VAR (ii) with only non-linear effects of uncertainty versus the model incorporating common stochastic volatility. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: $\widehat{Z}_{t-1} = ... = \widehat{Z}_{t-p} = 0$. Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: $\widehat{Z}_{t-1} > 0$ and $\widehat{Z}_{t-2} = ... = \widehat{Z}_{t-p} = 0$. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.

hit a local maximum. The responses of other macro variables in the VAR are virtually unchanged with the inclusion of the NFCI.

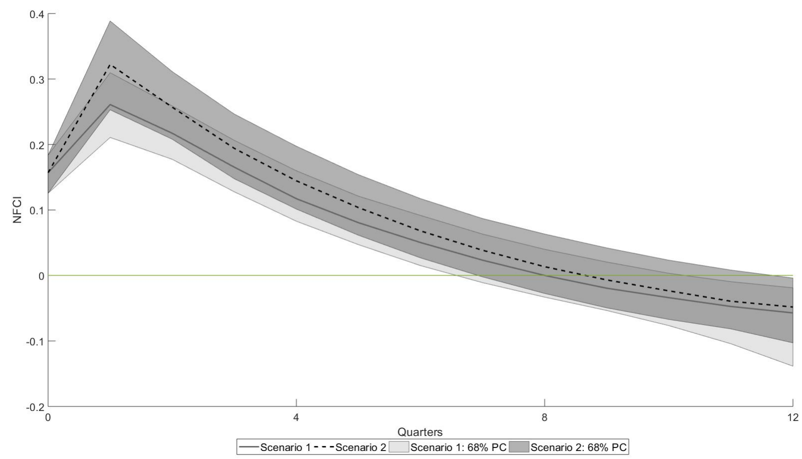

Figure 3: Iimpulse response of the Chicago Fed National Financial Conditions Index when added to the benchmark VAR (ii) with only non-linear effects of uncertainty. Scenario 1 represents times when the economy has not experienced a spike in uncertainty in recent history: $\widehat{Z}_{t-1} = ... = \widehat{Z}_{t-p} = 0$. Scenario 2 represents the case for which uncertainty has just reached a high level in the previous period: $\widehat{Z}_{t-1} > 0$ and $\widehat{Z}_{t-2} = ... = \widehat{Z}_{t-p} = 0$. We report cumulative impulse responses for those variables that enter the VAR in first differences of logs in order to interpret the effects on log-levels of these variables.

C Alternative Lag Length in the VAR

As discussed in Section 3.2, we considered lag orders from one to four for the VAR and found the BIC to favor one lag. Therefore, all of the results presented in the main text include one lag of all variables in the VAR. Given that we have quarterly data, we also looked more closely at the results when replicating all of our exercises for a VAR(4) instead. All of the qualitative conclusions are consistent across lag lengths: economic activity and prices contract more severely following a shock when uncertainty has recently been high. The relationships between the results suggested by our max uncertainty, linear, and threshold models are also comparable when increasing the lag length in the VAR. Finally, we generate similar conclusions regarding the role of the investment and consumption channels in the broader macroeconomy when facing heightened uncertainty.

References

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